Fluid flows analysis from image sequences

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Fluminance

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Fluid motion image analysis

Observation and analysis of flows from image sequences

- Environmental sciences (surveillance, forecasting, analysis of geophysical fluid)
- Hydrodynamic, aeronautic (turbulent wakes)
- Life sciences (bio-fluids)

Generic image analysis approaches inappropriate

Goals

Propose tools and models for the measurement, the analysis and the control of flows
## Fluid motion image analysis

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### Goals

Propose tools and models for the measurement, the analysis and the control of flows
Objective

Explore techniques to extract characteristic features of fluid flows along time

Axes of work

- **Estimation** of fluid flow velocity descriptors (reduced parametric or non-parametric representations of the flow)
- **Tracking** of salient fluid flows structures
- **Characterization** of reduced flow description
## Fluid flows velocity estimation

### Motion estimation problem

- Estimate \( v : \Omega \subset \mathbb{R}^2 \rightarrow v(x) = (u_x(x), u_y(x))^T \)
- From \( I : \Omega \times [0, T] \rightarrow I(x, t) \)

### Hypothesis

- Motion related to the photometric variations
- Motion field is spatially smooth

### Methods

- Discrete correlations
- Differential techniques
Correlation techniques

Principle

\[ \nu(x) = \arg \min_{\nu \in \{-U, \ldots, U\} \times \{-V, \ldots, V\}} \sum_{r \in \mathcal{W}(x)} C(l_2(r + \nu), l_1(r)) \]

- \( C \): squared difference or correlation function
- \( \nu \): discrete state space and rough spatial parameterization

Pro and Cons

- Fast (with FFT) local techniques
- Prone to erroneous spatial variabilities
- No spatial propagation of errors
- Difficult coupling with physical constraints
- Require non ambiguous photometric patterns
- Large scale measurements in practice
Differential techniques

**Principle**

\[ v(x) = \arg \min \int_{\Omega} \left\{ \left( \frac{dl}{dt} + f(l, v) \right)^2 + \lambda \left( g\left( \partial_{x_i y_j}^i+^j u_x, \partial_{x_i y_j}^i+^j u_y \right) \right) \right\} dx \]

- Functional gradient discretized (finite elements or finite differences)

**Pro and Cons**

- More general
- Theoretically finer spatial scales
- Propagation of errors
- Easy coupling with physical constraints
Differential techniques

Generic fluid motion estimator [Corpetti et al. PAMI 02, Yuan et al. JMIV 07]

- Data model: \[ \int_{\Omega} \left( \frac{dl}{dt} \right)^2 dx \]
- Smoothing function: \[ \int_{\Omega} (\| \nabla \text{curl} v \|^2 + \| \nabla \text{div} v \|^2) dx \]
- Mimetic finite differences
Differential techniques

Transmittance imagery fluid motion estimator [Corpetti et al. PAMI 02]

- Data model: \[ \int_{\Omega} \left( \frac{dl}{dt} + l \text{div} \nu \right)^2 dx \]
- Smoothing function: \[ \int_{\Omega} (\| \nabla \text{curl} \nu \|^2 + \| \nabla \text{div} \nu \|^2) dx \]

Airplane wing tip’s Vortex
Differential techniques

Transmittance imagery fluid motion estimator [Corpetti et al. PAMI 02]

- Data model: \( \int_{\Omega} \left( \frac{dI}{dt} + I \text{div} \mathbf{v} \right)^2 d\mathbf{x} \)

- Smoothing function: \( \int_{\Omega} (\| \nabla \text{curl} \mathbf{v} \|^2 + \| \nabla \text{div} \mathbf{v} \|^2) d\mathbf{x} \)

Vorticity map
Differential techniques

Transmittance imagery fluid motion estimator [Corpetti et al. PAMI 02]

- Data model: \( \int_{\Omega} \left( \frac{dl}{dt} + l\text{div}\nu \right)^2 dx \)

- Smoothing function: \( \int_{\Omega} (\|\nabla \text{curl}\nu\|^2 + \|\nabla \text{div}\nu\|^2) dx \)

Divergence map
Differential techniques

Schlieren motion estimator [Arnaud et al. ECCV 06]

- Data model: \[ \int_{\Omega} \left[ \frac{dl}{dt} + \frac{1}{2} l (\partial_x u_y + \partial_y u_x) \right]^2 dx \]

- Smoothing function: \[ \int_{\Omega} \| \nabla \text{curl} v \|^2 dx \]

- Additional constraint \( \text{div} v \simeq 0 \)
Differential techniques

Atmospheric motion layers estimator [Papadakis et al. TGRS 07]

- Data model: \[ \sum_k \int_{\Omega} \left[ \frac{dh^k}{dt} + h^k \text{div} v^k_{xy} - g \left( \rho^k u_z^k - \rho^{k+1} u_z^{k+1} \right) \right]^2 dx \]

- Smoothing function:
\[ \sum_k \int_{\Omega} \left( \| \nabla \text{curl} v^k_{xy} \|^2 + \| \nabla \text{div} v^k_{xy} \|^2 + \| \nabla u_z^k \|^2 \right) dx \]

Top of cloud pressure layers  Wind fields  Vert. wind. higher layer
Differential techniques

Low order parametric fluid motion estimator [Cuzol et al. IJCV 07]

- Data model: \( \int_{\Omega} (\nabla I^t v^\theta + \partial_t I)^2 dx \)

- Dedicated parametric representation:

\[
v^\theta(x) = \sum_i \gamma_i^{so} \nabla^\perp g_\sigma(x - x_i^{so}) + \sum_j \gamma_j^{ir} \nabla g_\sigma(x - x_j^{ir})
\]

particles
velocity
vorticity
Differential techniques

Low order parametric fluid motion estimator [Cuzol et al. IJCV 07]

- Data model: \[ \int_{\Omega} (\nabla I^t \nu^\theta + \partial_t I)^2 \, dx \]
- Dedicated parametric representation:

\[ \nu^\theta(x) = \sum_i \gamma_i^{so} \nabla^\perp g_{\sigma}(x - x_i^{so}) + \sum_j \gamma_j^{ir} \nabla g_{\sigma}(x - x_j^{ir}) \]
Differential techniques

Real experiments, wake flow at Re 3900 [Derrian et al. SSVM 11]
Fluid Motion estimation

Optical-flow versus PIV

- Results of similar quality on noise free particle images of 2D flow
- Physical constraints and dense representation
- No post processing

Black DNS; Red Corpetti-02; Blue Lavision (Davis 7.2); Green Yuan-07
## Fluid Motion estimation

### Optical-flow versus PIV

- Results of similar quality on noise free particle images of 2D flow
- Physical constraints and dense representation
- No post processing

### Common drawbacks

- No velocity measurements of the small scales
- No dynamical consistancy
- Difficult tuning of the smoothing parameter
Tracking of flow representations

Exploration of 2 methodological frameworks

**Stochastic filtering**
- Only adapted to reduced descriptors
- Recursive probabilistic frameworks
- Estimation of error covariance

**Variational assimilation**
- Well suited to high dimensional features
- Deterministic frameworks
- Batch processing
### Stochastic filtering in a non linear setting

#### Principle

- Given \( dx_t = M(x_t)dt + \sigma(t)dB_t \) and \( y_k = H(x_k) + \gamma_k \)
- Estimate the pdf \( p(x_{0:k}|y_{1:k}) = p(x_{0:k-1}|y_{1:k-1}) \frac{p(y_k|x_{t=k})p(x_{t=k}|x_{k-1})}{p(y_k|y_{1:k-1})} \)

#### Gaussian linear model: Kalman Filtering

\[
\begin{align*}
    &\mathbb{E}(x_k|y_{1:k}) = x_k^a = x_{k|k-1} + K(y_t - Hx_{k|k-1}), \\
    &K = \Sigma_{k|k-1}H^T(H\Sigma_{k|k-1}H^T + R)^{-1} \\
    &\mathbb{E}((x_k - x_k^a)(x_k - x_k^a)^T|y_{1:k}) = P_k^a = (I - KH)\Sigma_{k|k-1}
\end{align*}
\]

#### Non Linear dynamics, linear measure: Ensemble Kalman Filtering

Kalman updates computed from a set of samples \( x_t^{(i)}, i = 1, \ldots, N \)
Particle Filter

Non-linear dynamics and observations

- \( p(x_k|z_{1:k}) \approx \sum_i w_k^{(i)} \delta_{x_k^{(i)}} \)
- prediction step (importance distribution sampling \( \pi \))

\[
x_k^{(i)} \sim \pi(x_{0:k}|y_{1:k}) = \pi(x_{0:k-1}|y_{1:k-1})\pi(x_k|y_{1:k}, x_{0:k-1})
\]
- correction step

\[
w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(y_k|x_k^{(i)})p(x_k^{(i)}|x_{k-1}^{(i)})}{\pi(x_k^{(i)}|x_{0:k-1}^{(i)}, y_{1:k})}
\]
Ensemble Kalman filter extension

Importance distribution

- Bootstrap filter

\[
\pi(x_k|x_{0:k-1}^{(i)}, y_{1:t}) = p(x_k|x_{k-1}^{(i)}) \Rightarrow w_k^{(i)} \propto w_{k-1}^{(i)} p(y_k|x_k^{(i)})
\]

⇒ strong limitation in high dimensional space

- Ensemble Kalman proposal distribution (Papadakis et al. Tellus 10)

\[
\pi(x_k|x_{0:k-1}^{(i)}, y_{1:k}) = p(x_k|x_{k-1}^{(i)}, y_k) \approx \mathcal{N}(x_k^{a}, (I - K^e H)\Sigma_k|_{k-1})
\]

\[
w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(y_k|x_k^{(i)})p(x_k^{(i)}|x_{k-1}^{(i)})}{\mathcal{N}(x_k^{(i)} - x_k^{a}; 0, P_k^{a})}
\]

where with linear observation operator

\[
(N-1)P_k^a = x_k^f x_k^f T - x_k^f x_k^f T H^T (H x_k^f x_k^f T H^T + \tilde{R})^{-1} H x_k^f x_k^f T
\]
Ensemble Kalman filter extension

Importance distribution

- Bootstrap filter

\[ \pi(x_k|x_{0:k-1}^{(i)}, y_{1:t}) = p(x_k|x_{k-1}^{(i)}) \Rightarrow w_k^{(i)} \propto w_{k-1}^{(i)} p(y_k|x_k^{(i)}) \]

\( \Rightarrow \) strong limitation in high dimensional space

- Ensemble Kalman proposal distribution (Papadakis et al. Tellus 10)

\[ \pi(x_k|x_{0:k-1}^{(i)}, y_{1:k}) = p(x_k|x_{k-1}^{(i)}, y_k) \approx \mathcal{N}(x_k^a, (I - K^eH)\Sigma_e^k x_{k-1}^{(i)}) \]

\[ w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(y_k|x_k^{(i)})p(x_k^{(i)}|x_k^{(i)})}{\mathcal{N}(x_k^{(i)} - x_k^a; 0, P_k^a)} \]

with nonlinear observation operator (Beyou et al. Tellus 13)

\[ (N - 1)P_k^a = x_k^f x_k^f^T - x_k^f H(x_k^f)^T (H(x_k^f)H(x_k^f)^T + \tilde{R})^{-1} H(x_k^f)x_k^f^T \]
Vorticity recovering from image data

**Experiments: 2D velocity-vorticity**

- **Dynamics**
  \[
  d\xi + \nabla \xi \cdot \mathbf{w} \, dt = \frac{1}{Re} \Delta \xi \, dt + \sigma_Q \, dW,
  \]

- \(dW\) isotropic Gaussian field
  \[
  Q(r, \tau) = \mathbb{E}(dW(x, t)dW(x + r, t + \tau)) = g_\lambda(r)dt\delta(\tau),
  \]

\[
\sigma_Q = 0.1 \quad \sigma_B = 1 \quad \sigma_Q = 0.1 \quad \sigma_B = 0.5 \quad \sigma_Q = 0.1 \quad \sigma_B = 0.1
\]
Vorticity recovering from image data

Filtering system

- Velocity-vorticity stochastic formulation

\[ d\xi + \nabla \xi \cdot w \, dt = \frac{1}{Re} \Delta \xi \, dt + \sigma_Q \, dW, \]

- Measurements

1) Local motion measurements

\[ y_k = w + \gamma_k \]

2) Image reconstruction error

\[ I(x, k) = I(x + d_{k+1}(x), k + 1) + \eta_k \]
Results: 2D DNS sequence

passive scalar

vorticity
Results: 2D DNS sequence

RMSE vorticity

Energy Spectrum
Results: 2D DNS sequence

Filtering results (initialization with local motion estimates)
Results: Oceanic SST images
Stochastic filtering for curve tracking

Curve tracking

- **Objective**: track the evolution of a 2D closed curve in the image domain
- **Difficulty**: evolution model not accessible (projection of a moving 3D curve)
- \( \Rightarrow \) stochastic dynamics inferred from the data with a low dimensional noise
Stochastic filtering for curve tracking

Curve tracking (Avenel et al. JMMIV 14)

- Curve described through an implicit function $\varphi$
  $$C_t = \{x | \varphi(x, t) = 0\}$$
  $\Rightarrow$ The curve is specified as the zero level set of $\varphi$

- Curve dynamics driven by the data and low dimensional noise
  $$dC_t = w_n nt + \sigma_1 n dB_t^n + \sigma_2 n^\perp dB_t^\perp$$

- Deformation field: extension to the whole plane of the curve evolution
  $$dX_t = w_n^* \frac{\nabla \varphi}{|\nabla \varphi|} dt + \sigma_n \frac{\nabla \varphi}{|\nabla \varphi|} dB_t^n + \sigma_{\tau} \frac{\nabla \varphi^\perp}{|\nabla \varphi|} dB_t^\perp$$

- Surface $\varphi$ transported by the curve deformation field:
  $$d\varphi(t, x) = 0$$
  $\Rightarrow$ Require the computation of $d\varphi(X_t, t)$
Differential of the implicit surface

- Surface $\varphi$ is necessarily a stochastic process
  For a fixed point $y$, $\varphi$ solution of

  $$d\varphi_t(y) = b(y, t)dt + f(y, t)dB_{n,t} + g(y, t)dB_{\tau,t},$$

- Use of Ito-Wentzell formula (differential of $\varphi \circ X_t$)

  $$d\varphi(x, t) = d\varphi_t(x) + \nabla \varphi^T dX_t + \frac{1}{2} \sum_{i,j} d\left\langle X^i_t, X^j_t \right\rangle \frac{\partial^2 \varphi}{\partial x_i \partial x_j}$$

  $$+ \sum_i d\left\langle \frac{\partial \varphi}{\partial x_i}, \left. X^i_t \right\rangle_t = 0$$
Differential of the implicit surface

- Surface $\varphi$ is necessarily a stochastic process
  For a fixed point $y$, $\varphi$ solution of

  $$
  d\varphi_t(y) = b(y, t)dt + f(y, t)dB_{n,t} + g(y, t)dB_{\tau,t},
  $$

- Use of Ito-Wentzell formula (differential of $\varphi \circ X_t$)

  $$
  d\varphi_t(x) = -\nabla\varphi^T w_n^* dt - \frac{\sigma_n^2}{2} dt \left( \Delta \varphi - \frac{1}{|\nabla \varphi|^2} \nabla \varphi^T \nabla^2 \varphi \nabla \varphi \right) + \frac{\sigma_n^2}{2} \left( \frac{1}{|\nabla \varphi|^2} \nabla \varphi^T \nabla^2 \varphi \nabla \varphi \right) - \sigma_n |\nabla \varphi| dB_{n,t},
  $$
Stochastic filtering for curve tracking

Definition of the transportation motion field

- Infer directly the velocity from each particle displacements
- Adjunction of a new vectorial level set $\psi$ representative of the grid coordinates at the previous time transported by the curve

\[ \psi^k(x, k - 1) = x \]

\( \psi^k(x, t) \Rightarrow \) coordinates of point \( x \) at time \( k - 1 \) for \( t \in [k - 1, k] \)

- Ito-Wentzell formula for the differential of \( \psi^k \)

\[
d\psi^i_t(x) = -\left(\nabla \psi^i_t\right)^T v^*_n dt \\
- \left(\nabla \psi^i_t\right)^T \left( \frac{\nabla \varphi}{|\nabla \varphi|} \sigma_n dB_{n,t} + \frac{\nabla \varphi}{|\nabla \varphi|} \sigma_\tau dB_{\tau,t} \right) \\
- A_i dt + \frac{\sigma_n F_i}{|\nabla \varphi|} dt + \frac{\sigma_\tau G_i}{|\nabla \varphi|}. \]
Definition of the transportation motion field

- Transportation motion field

\[ v^*(x, t)dt = \frac{1}{\Delta t}(x - E(\psi^k(x, k)|C_k)), \forall t \in [k, k + 1], \]

Considering the particle approximation this velocity field is computed as:

\[ v^*(x, t)dt = \frac{1}{\Delta t} \left( x - \frac{1}{N} \sum_{i=1}^{N} w^{(i)} \psi^{k,(i)}(x, k) \right), \tag{1} \]

for \( t \in [k, k + 1] \), with \( \psi^{k,(i)} \) auxiliary function associated to particle \( \varphi_k^{(i)} \) and \( w_k^{(i)} \) its importance weight.
Likelihood definition

Likelihood that depends on the similarity between photometric distributions inside the curve at times $t = 0$ and $t = k$.

$$p(y_t | C_t^{(i)}) \propto \exp\{-\lambda d(h_0, h_k^{(i)})\}, \quad (2)$$

$d$: Hellinger distance between $h_0$ the reference interior histogram at time 0 and $h_k^{(i)}$ the histogram associated to the $i$-th surface sample at time $k$

$$d(p, q) = \left(1 - \sum_{j \in X} \sqrt{p(j)q(j)}\right)^{1/2}.$$
Stochastic filtering for curve tracking

Results

Convective cell tracking
Convective cell: velocity field applied
### Results

Convective cell: curve points trajectories
Stochastic filtering for curve tracking
### Variational assimilation

#### Principle

- **Initial condition + dynamical law**

\[
\frac{\partial \mathbf{x}}{\partial t}(x, t) + \mathbb{M}(\mathbf{x}(x, t)) = p(x, t)
\]  
\[\mathbf{x}(x, t_0) = \mathbf{x}_0(x) + \epsilon_n(x),\]  

- **+ observations**

\[
\mathbf{y}(x, t) = \mathbb{H}(\mathbf{x}(x, t)) + \epsilon_o(x, t)
\]

\(\mathbf{x}_0\) initial condition \((t_0)\) and \((p(t), \epsilon_n)\) control variables.

- **Minimize w.r.t. \((p, \epsilon_n)\)**

\[
J(p, \epsilon_n) = \int_{t_0}^{t_f} \left\| \mathbf{y}(x, t) - \mathbb{H}(\mathbf{x}(x, t)) \right\|_R^2 + \int_{t_0}^{t_f} \left\| p(x, t) \right\|_Q^2 + \left\| \epsilon_n \right\|_B^2
\]
Variational assimilation

Functional gradient

- Introduction of an adjoint variable

\[
\begin{align*}
- \frac{\partial \lambda}{\partial t} (t) + (\partial_x M)^* \lambda(t) &= (\partial_x H)^* R^{-1} (y - H(x))(t) \\
\lambda(t_f) &= 0,
\end{align*}
\]

- Functional gradient

\[
\begin{align*}
\partial_p J &= Q^{-1} (\partial_t x + M(x)) - \lambda, \\
\partial_{\epsilon_n} J &= \lambda(t_0) + B^{-1} (x(t_0) - X_0)
\end{align*}
\]
Variational assimilation for atmospheric layer motion tracking (Papadakis 07, Corpetti et al. Tellus 09)

Context: multi-layer imagery

- Sparse images (clouds), inaccurate vertical coordinates

Layer surfaces $s^k$

- cloud classified in $k-th$ layer
- $s^k$ defined by clouds altitude in $C^k$

Sparse pressure differences $h_{obs}^k$

$$h_{obs}^k = \begin{cases} 
\bar{p}(s^k) - p(x, y, s^{k+1}) & \text{if } (x, y) \in C^{k+1} \\
0 & \text{else}
\end{cases}$$
Variational assimilation for atmospheric layer motion tracking (Papadakis 07, Corpetti et al. Tellus 09)

Context: multi-layer imagery

- Sparse images (clouds), inaccurate vertical coordinates

![EUMETSAT](image1)
![EUMETSAT](image2)

Sparse images \( (h_{obs}^1, h_{obs}^2, h_{obs}^3) \) for a 3 layer decomposition

Layer surfaces \( s^k \)
- Cloud classified in \( k^{th} \) layer
- \( s^k \) defined by clouds altitude in \( C^k \)

Sparse pressure differences \( h_{obs}^k \)

\[
h_{obs}^k = \begin{cases} 
\bar{p}(s^k) - p(x, y, s^{k+1}) & \text{if } (x, y) \in C^{k+1} \\
0 & \text{else.}
\end{cases}
\]
Variational assimilation for atmospheric layer motion tracking (Papadakis 07, Corpetti et al. Tellus 09)

Example of pressure differences image observations $h^k_{obs}$
Variational assimilation for atmospheric layer motion tracking (Papadakis 07, Corpetti et al. Tellus 09)

Dynamical model: simplified divergence and vorticity multi-layer shallow water model

\[
\begin{cases}
\omega_t^k + \mathbf{v}^k \cdot \nabla \omega^k + (\omega^k + f\phi)D^k - \nu_T \Delta \omega^k = \nu_1(D^k) \\
D_t^k + \mathbf{v}^k \cdot \nabla D^k + (D^k)^2 - \nu_T \Delta D^k = \nu_2(D^k, h^k)
\end{cases}
\]

Motion field from Biot Savart law

\[
\mathbf{v}^k = \nabla_{\perp} G * \omega^k + \nabla G * D^k + \mathbf{v}_\text{sol}^k + \mathbf{v}_\text{irr}^k + \mathbf{v}_\text{har}^k \\
= \left[ \nabla_{\perp} G*, \nabla G* \right]_{\mathbb{H}_G} \begin{bmatrix} \omega^k \\ D^k \end{bmatrix} \text{ state variable } \mathbf{x} + \mathbf{v}_\text{har}^k
\]
Variational assimilation for atmospheric layer motion tracking (Papadakis 07, Corpetti et al. Tellus 09)

Observations: optical-flow constraint equation (OFCE)

- Shallow water mass conservation

\[
\frac{\partial h^k_{\text{obs}}}{\partial t} + \nabla h^k_{\text{obs}} \cdot \mathbf{v}^k + h^k_{\text{obs}} \text{div} \mathbf{v}^k \approx 0
\]

- Constant field \( \mathbf{v}^k \) within a spatial neighborhood

\( (K_{\delta x} : \text{Gaussian kernel}) \)

\[
K_{\delta x} \ast \left( \frac{\partial h^k_{\text{obs}}}{\partial t} + \nabla h^k_{\text{obs}} \cdot \mathbf{v}^k + h^k_{\text{obs}} \text{div} \mathbf{v}^k \right) \approx 0,
\]

i.e. observation operator definition \( \mathbf{y} = \mathbb{H}(\mathbf{x}) \) with \( \mathbf{v}^k = \mathbb{H}_G \mathbf{x} \)

(\( \mathbf{x} \) is vorticity and divergence) :

\[
\begin{align*}
\mathbf{y} &= K_{\delta x} \ast \frac{\partial h^k_{\text{obs}}}{\partial t} \\
\mathbb{H} &= - (K_{\delta x} \ast \nabla h^k_{\text{obs}})^T \mathbb{H}_G - (K_{\delta x} \ast h^k_{\text{obs}}) [1 \ 0]
\end{align*}
\]
Experiments on METEOSAT image sequence

estimates (wind, vorticity & divergence of intermediate layer)

comparison with frame-to-frame motion estimation (vorticity & divergence)
Experiments on METEOSAT image sequence

estimates (wind, vorticity & divergence of lower layer)

estimates (wind, vorticity & divergence of higher layer)
Real sequence: vince (1/4)

- Cyclone (9th October 2005)
- Infrared data
- Sequence of 116 images
Real sequence: vince (2/4)

- Cyclone (9th October 2005)
- Visible data
- Sequence of 116 images
Real sequence: vince (3/4)

- Cyclone (9\textsuperscript{th} october 2005)
- Velocities obtained with the visible channel superimposed to IR data
- Sequence of 116 images
Real sequence: vince (4/4)

- Comparison with fluid dedicated optical-flow estimator (without temporal consistency)

Assimilated vorticity

No temporal consistency
## Data assimilation

### Stochastic filtering
- Recursive technique
- Probability distribution
- Adequate stochastic formulation of the dynamics
- Appropriate noise modeling

### Variational assimilation
- Deterministic batch framework
- Adapted to state space of great dimension
- Require to built the adjoint of the tangent linear dynamics
- Intrinsic linearization
Conclusions

What did we do?
- Explorations of tracking / assimilation techniques for time resolved flow images
- Very good performances for direct methods
- Process for “learning” dynamics from image data
- Specification of dynamics under uncertainty

But how far can we go?
- Experiments mainly on 2D or 2D 1/2 flows
- Necessity to go toward 3D and more complex dynamics
- May have big potential applications for geo-physical applications