Fluid flows analysis from image sequences

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Fluid motion image analysis

Observation and analysis of flows from image sequences

- Environmental sciences (surveillance, forecasting, analysis of geophysical fluid)
- Hydrodynamic, aeronautic (turbulent wakes)
- Life sciences (bio-fluids)

Generic image analysis approaches inappropriate

Goals

Propose tools and models for the measurement, the analysis and the control of flows

Fluid motion image analysis

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Propose tools and models for the measurement, the analysis and the control of flows

Objective

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- Explore techniques to extract characteristic features of fluid flows along time

Axes of work

- Estimation of fluid flow velocity descriptors (reduced parametric or non parametric representations of the flow)
- Tracking of salient fluid flows structures
- Characterization of reduced flow description

Fluid flows velocity estimation

Motion estimation problem

- Estimate $v\colon \Omega \subset \mathbb{R}^2 \to v(x) = \big(u_x(x), u_y(x)\big)^\top$
- From $I\colon \Omega \times [0, T] \to I(x, t)$

Hypothesis

- Motion related to the photometric variations
- Motion field is spatially smooth

Methods

- Discrete correlations
- Differential techniques

Correlation techniques

Principle

$$v(x) = \arg \min_{v \in \{-U,\ldots,U\} \times \{-V,\ldots,V\}} \sum_{r \in \mathcal{W}(x)} \mathcal{C}(I_2(r+v), I_1(r))$$

- $\mathcal{C}$: squared difference or correlation function
- v: discrete state space and rough spatial parameterization

Pro and Cons

- Fast (with FFT) local techniques
- Prone to erroneous spatial variabilities
- No spatial propagation of errors
- Difficult coupling with physical constraints
- Require non ambigous photometric patterns
- Large scale measurements in practice

Differential techniques

Principle

$$v(x) = \arg\min \int_\Omega \left\{ \left(\frac{dI}{dt} + f(I,v)\right)^2 + \lambda\Big(g(\partial^{i+j}_{x^i y^j} u_x, \partial^{i+j}_{x^i y^j} u_y)\Big) \right\} dx$$

- Functional gradient discretized (finite elements or finite differences)

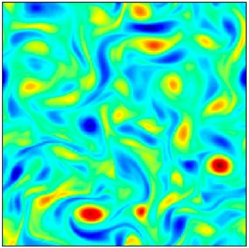
Pro and Cons

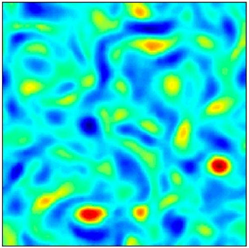
- More general
- Theoretically finer spatial scales
- Propagation of errors
- Easy coupling with physical constraints

Differential techniques

Generic fluid motion estimator [Corpetti et al. PAMI 02, Yuan et al. JMIV 07]

- Data model: $\int_{\Omega} (\frac{dI}{dt})^2 dx$
- Smoothing function: $\int_{\Omega} (\|\nabla \mathrm{curl} v\|^2 + \|\nabla \mathrm{div} v\|^2) dx$
- Mimetic finite differences





Differential techniques

Transmittance imagery fluid motion estimator [Corpetti et al. PAMI 02]

- Data model: $\int_{\Omega}(\frac{dI}{dt} + I\mathrm{div}v)^2\mathrm{d}x$
- Smoothing function: $\int_{\Omega}(\|\nabla\mathrm{curl}v\|^2 + \|\nabla\mathrm{div}v\|^2)dx$

Airplane wing tip's Vortex

Differential techniques

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Vorticity map

Differential techniques

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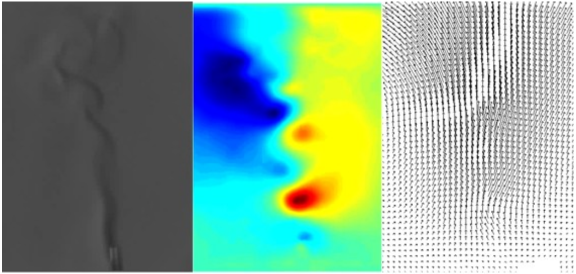
- Smoothing function: $\int_\Omega (\|\nabla \mathrm{curl} v\|^2 + \|\nabla \mathrm{div} v\|^2) dx$

Divergence map

Differential techniques

Schlieren motion estimator [Arnaud et al. ECCV 06]

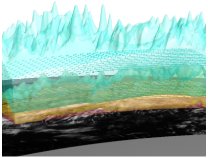
- Data model: $\int_\Omega [\frac{dI}{dt} + \frac{1}{2} I (\partial_x u_y + \partial_y u_x)]^2 dx$
- Smoothing function: $\int_\Omega \|\nabla \text{curl} v\|^2 dx$
- Additional constraint div$v \simeq 0$



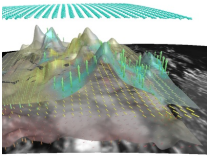
Differential techniques

Atmospheric motion layers estimator [Papadakis et al. TGRS 07]

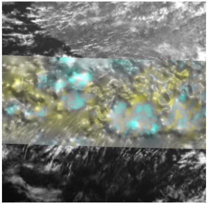
- Data model: $\sum_k \int_\Omega [\frac{dh^k}{dt} + h^k \mathrm{div} v^k_{xy} - g(\rho^k u^k_z - \rho^{k+1} u^{k+1}_z)]^2 dx$
- Smoothing function:
$\sum_k \int_\Omega (\|\nabla \mathrm{curl} v^k_{xy}\|^2 + \|\nabla \mathrm{div} v^k_{xy}\|^2 + \|\nabla u^k_z\|^2) dx$



Top of cloud pressure layers



Wind fields



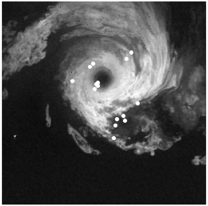
Vert. wind. higher layer

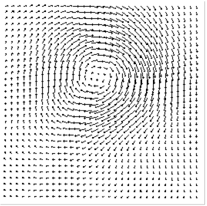
Differential techniques

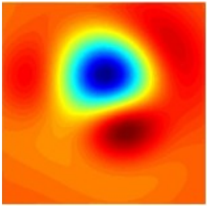
Low order parametric fluid motion estimator [Cuzol et al. IJCV 07]

- Data model: $\int_\Omega (\nabla I^t v^\theta + \partial_t I)^2 dx$
- Dedicated parametric representation:

$$v^\theta(x) = \sum_i \gamma_i^{so} \nabla^\perp g_\sigma(x - x_i^{so}) + \sum_j \gamma_j^{ir} \nabla g_\sigma(x - x_j^{ir})$$


particles


velocity


vorticity

Differential techniques

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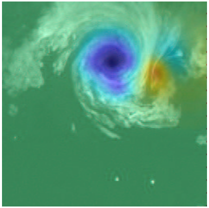
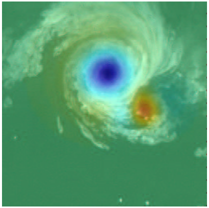


#1



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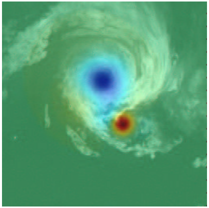


#3

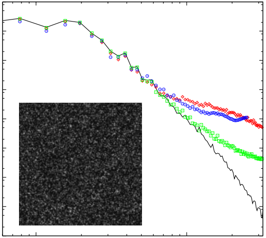
Differential techniques

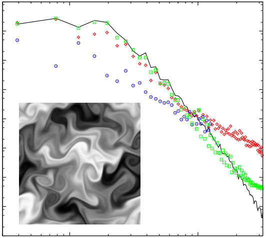
Real experiments, wake flow at Re 3900 [Derrian et al. SSVM 11]

Fluid Motion estimation

Optical-flow versus PIV

- Results of similar quality on noise free particle images of 2D flow
- Physical constraints and dense representation
- No post processing





Black DNS; Red Corpetti-02; Blue Lavision (Davis 7.2); Green Yuan-07

Fluid Motion estimation

Optical-flow versus PIV

- Results of similar quality on noise free particle images of 2D flow
- Physical constraints and dense representation
- No post processing

Common drawbacks

- No velocity measurements of the small scales
- No dynamical consistancy
- Difficult tuning of the smoothing parameter

Tracking of flow representations

Exploration of 2 methodological frameworks

Stochastic filtering

- Only adapted to reduced descriptors
- Recursive probabilistic frameworks
- Estimation of error covariance

Variational assimilation

- Well suited to high dimensional features
- Deterministic frameworks
- Batch processing

Stochastic filtering in a non linear setting

Principle

- Given $d\mathbf{x}_t = \mathbf{M}(\mathbf{x}_t)dt + \sigma(t)d\mathbf{B}_t$ and $\mathbf{y}_k = \mathbf{H}(\mathbf{x}_k) + \gamma_k$
- Estimate the pdf $p(\mathbf{x}_{0:k}|\mathbf{y}_{1:k}) = p(\mathbf{x}_{0:k-1}|\mathbf{y}_{1:k-1})\frac{p(\mathbf{y}_k|\mathbf{x}_{t=k})p(\mathbf{x}_{t=k}|\mathbf{x}_{k-1})}{p(\mathbf{y}_k|\mathbf{y}_{1:k-1})}$

Gaussian linear model: Kalman Filtering

$$\mathbb{E}(\mathbf{x}_k|\mathbf{y}_{1:k}) = \mathbf{x}_k^a = \mathbf{x}_{k|k-1} + \mathbf{K}(\mathbf{y}_t - \mathbf{H}\mathbf{x}_{k|k-1}),$$
$$\mathbf{K} = \mathbf{\Sigma}_{k|k-1}\mathbf{H}^T(\mathbf{H}\mathbf{\Sigma}_{k|k-1}\mathbf{H}^T + \mathbf{R})^{-1}$$
$$\mathbb{E}((\mathbf{x}_k - \mathbf{x}_k^a)(\mathbf{x}_k - \mathbf{x}_k^a)^T|\mathbf{y}_{1:k}) = \mathbf{P}_k^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\Sigma_{k|k-1}$$

Non Linear dynamics, linear measure: Ensemble Kalman Filtering

Kalman updates computed from a set of samples $\mathbf{x}_t^{(i)},\ i = 1, \ldots, N$

Particle Filter

Non-linear dynamics and observations

- $p(\mathbf{x}_k|\mathbf{z}_{1:k}) \simeq \sum_i w_k^{(i)} \delta_{\mathbf{x}_k^{(i)}}$
- prediction step (importance distribution sampling π)

$$\mathbf{x}_k^{(i)} \sim \pi(\mathbf{x}_{0:k}|\mathbf{y}_{1:k}) = \pi(\mathbf{x}_{0:k-1}|\mathbf{y}_{1:k-1})\pi(\mathbf{x}_k|\mathbf{y}_{1:k}, \mathbf{x}_{0:k-1})$$

- correction step

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(\mathbf{y}_k|\mathbf{x}_k^{(i)})p(\mathbf{x}_k^{(i)}|\mathbf{x}_{k-1}^{(i)})}{\pi(\mathbf{x}_k^{(i)}|\mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{1:k})}$$

Ensemble Kalman filter extension

Importance distribution

- Bootstrap filter

$$\pi(\mathbf{x}_k|\mathbf{x}^{(i)}_{0:k-1}, \mathbf{y}_{1:t}) = p(\mathbf{x}_k|\mathbf{x}^{(i)}_{k-1}) \Rightarrow w^{(i)}_k \propto w^{(i)}_{k-1} p(\mathbf{y}_k|\mathbf{x}^{(i)}_k)$$

$\Rightarrow$ strong limitation in high dimensional space

- Ensemble Kalman proposal distribution (Papadakis et al. Tellus 10)

$$\pi(x_k|x^{(i)}_{0:k-1}, \mathbf{y}_{1:k}) = p(x_k|x^{(i)}_{k-1}, \mathbf{y}_k) \approx \mathcal{N}(x^a_k, (\mathbb{I} - \mathbf{K}^e\mathbf{H})\mathbf{\Sigma}^e_{k|k-1})$$

$$w^{(i)}_k \propto w^{(i)}_{k-1} \frac{p(\mathbf{y}_k|\mathbf{x}^{(i)}_k) p(\mathbf{x}^{(i)}_k|\mathbf{x}^{(i)}_{k-1})}{\mathcal{N}\left(\mathbf{x}^{(i)}_k - x^a_k; 0, \mathbf{P}^a_k\right)}$$

where with linear observation operator

$$(N-1)\mathbf{P}^a_k = \mathbf{x}^f_k {\mathbf{x}^f_k}^T - \mathbf{x}^f_k {\mathbf{x}^f_k}^T \mathbf{H}^T (\mathbf{H}\mathbf{x}^f_k {\mathbf{x}^f_k}^T \mathbf{H}^T + \tilde{\mathbf{R}})^{-1} \mathbf{H}\mathbf{x}^f_k {\mathbf{x}^f_k}^T$$

Ensemble Kalman filter extension

Importance distribution

- Bootstrap filter

$$\pi(\mathbf{x}_k|\mathbf{x}_{0:k-1}^{(i)},\mathbf{y}_{1:t}) = p(\mathbf{x}_k|\mathbf{x}_{k-1}^{(i)}) \Rightarrow w_k^{(i)} \propto w_{k-1}^{(i)} p(\mathbf{y}_k|\mathbf{x}_k^{(i)})$$

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$$\pi(x_k|x_{0:k-1}^{(i)},\mathbf{y}_{1:k}) = p(x_k|x_{k-1}^{(i)},\mathbf{y}_k) \approx \mathcal{N}(x_k^a,(\mathbb{I}-\mathbf{K}^e\mathbf{H})\mathbf{\Sigma}_{k|k-1}^e)$$

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(\mathbf{y}_k|\mathbf{x}_k^{(i)})p(\mathbf{x}_k^{(i)}|\mathbf{x}_{k-1}^{(i)})}{\mathcal{N}\left(\mathbf{x}_k^{(i)}-x_k^a;0,\mathbf{P}_k^a\right)}$$

with nonlinear observation operator (Beyou et al. Tellus 13)

$$(N-1)\mathbf{P}_k^a = \mathbf{x}_k^f {\mathbf{x}_k^f}^T - \mathbf{x}_k^f \mathbf{H}(\mathbf{x}_k^f)^T(\mathbf{H}(\mathbf{x}_k^f)\mathbf{H}(\mathbf{x}_k^f)^T+\tilde{\mathbf{R}})^{-1}\mathbf{H}(\mathbf{x}_k^f){\mathbf{x}_k^f}^T$$

Vorticity recovering from image data

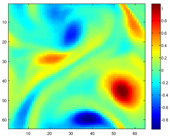
Experiments: 2D velocity-vorticity

- Dynamics

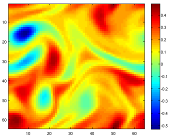
$$d\xi + \nabla\xi \cdot \mathbf{w} dt = \frac{1}{Re}\Delta\xi dt + \sigma_Q dW,$$

- dW isotropic Gaussian field

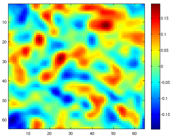
$$Q(\mathbf{r},\tau) = \mathbb{E}(dW(\mathbf{x},t)dW(\mathbf{x}+\mathbf{r},t+\tau)) = g_\lambda(\mathbf{r})dt\delta(\tau),$$



$\sigma_Q = 0.1$ $\sigma_B = 1$



$\sigma_Q = 0.1$ $\sigma_B = 0.5$



$\sigma_Q = 0.1$ $\sigma_B = 0.1$

Vorticity recovering from image data

Filtering system

- Velocity-vorticity stochastic formulation

$$d\xi + \nabla\xi \cdot \mathbf{w} dt = \frac{1}{Re}\Delta\xi dt + \sigma_{Q} dW,$$

- Measurements

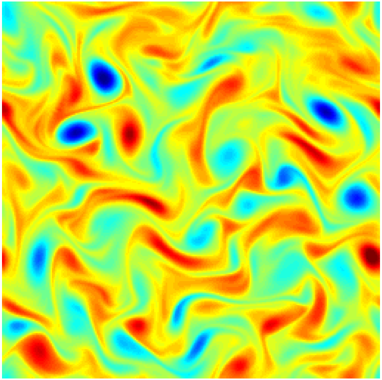
1) Local motion mesurements

$$\mathbf{y}_k = \mathbf{w} + \gamma_k$$

2) Image reconstruction error

$$I(x,k) = I(x + \mathbf{d}_{k+1}(\mathbf{x}), k+1) + \boldsymbol{\eta}_k$$

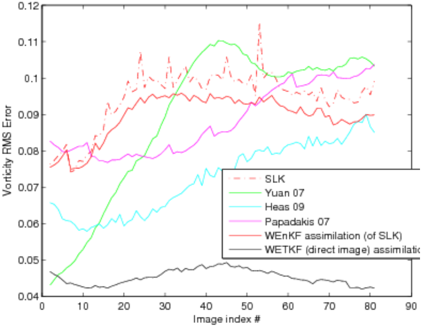
Results: 2D DNS sequence



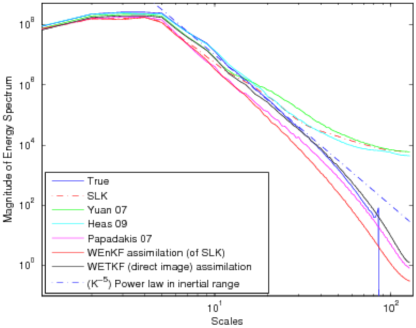
passive scalar

vorticity

Results: 2D DNS sequence



RMSE vorticity



Energy Spectrum

Results: 2D DNS sequence

Filtering results (initialization with local motion estimates)

Results: Oceanic SST images

Stochastic filtering for curve tracking

Curve tracking

- Objective: track the evolution of a 2D closed curve in the image domain
- Difficulty: evolution model not accessible (projection of a moving 3D curve)
- $\Rightarrow$ stochastic dynamics infered from the data with a low dimensional noise

Stochastic filtering for curve tracking

Curve tracking (Avenel et al. JMIV 14)

- Curve described through an implicit function φ

$$\mathcal{C}_t = \{\mathbf{x} | \varphi(\mathbf{x}, t) = 0\}$$

$\Rightarrow$ The curve is specified as the zero level set of φ

- Curve dynamics driven by the data and low dimensional noise

$$d\mathcal{C}_t = w_n \mathbf{n} dt + \sigma_1 \mathbf{n} dB_t^n + \sigma_2 \mathbf{n}^\perp dB_t^\tau$$

- Deformation field: extension to the whole plane of the curve evolution

$$d\mathbf{X}_t = w_n^* \frac{\boldsymbol{\nabla}\varphi}{|\boldsymbol{\nabla}\varphi|} dt + \sigma_n \frac{\boldsymbol{\nabla}\varphi}{|\boldsymbol{\nabla}\varphi|} dB_t^n + \sigma_\tau \frac{\boldsymbol{\nabla}\varphi^\perp}{|\boldsymbol{\nabla}\varphi|} dB_t^\tau$$

- Surface φ transported by the curve deformation field:

$$d\varphi(t, x) = 0$$

$\Rightarrow$ Require the computation of $d\varphi(X_t, t)$

Stochastic filtering for curve tracking

Differential of the implicit surface

- Surface φ is necessarily a stochastic process
  For a fixed point y, φ solution of

$$d\varphi_t(y) = b(y,t)dt + f(y,t)dB_{n,t} + g(y,t)dB_{\tau,t},$$

- Use of Ito-Wentzell formula (differential of $\varphi \circ X_t$)

$$d\varphi(x,t) = d\varphi_t(x) + \nabla\varphi^T dX_t + \frac{1}{2}\sum_{i,j} d\left\langle X_t^i, X_t^j \right\rangle \frac{\partial^2\varphi}{\partial x_i \partial x_j}$$

$$+ \sum_i d\left\langle \frac{\partial\varphi}{\partial x_i}, X_t^i \right\rangle_t = 0$$

Stochastic filtering for curve tracking

Differential of the implicit surface

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$$d\varphi_t(y) = b(y,t)dt + f(y,t)dB_{n,t} + g(y,t)dB_{\tau,t},$$

- Use of Ito-Wentzell formula (differential of $\varphi \circ X_t$)

$$\begin{aligned} d\varphi_t(x) &= -\nabla\varphi^T w_n^* dt - \frac{\sigma_\tau^2 dt}{2}(\Delta\varphi - \frac{1}{|\nabla\varphi|^2}\nabla\varphi^T\nabla^2\varphi\nabla\varphi) \\ &+ \frac{\sigma_n^2 dt}{2}\left(\frac{1}{|\nabla\varphi|^2}\nabla\varphi^T\nabla^2\varphi\nabla\varphi\right) - \sigma_n|\nabla\varphi|dB_{n,t}, \end{aligned}$$

Stochastic filtering for curve tracking

Definition of the transportation motion field

- Infer directly the velocity from each particle displacements
- Adjunction of a new vectorial level set $\boldsymbol{\psi}$ representative of the grid coordinates at the previous time transported by the curve

$$\boldsymbol{\psi}^k(x, k-1) = x$$

$\boldsymbol{\psi}^k(x, t) \Rightarrow$ coordinates of point x at time $k-1$ for $t \in [k-1, k]$

- Ito-Wentzell formula for the differential of $\boldsymbol{\psi}^k$

$$\begin{aligned} d\psi_t^i(x) = &-(\nabla\psi_t^i)^T v_n^* dt \\ &- (\nabla\psi_t^i)^T \left(\frac{\nabla\varphi}{|\nabla\varphi|}\sigma_n dB_{n,t} + \frac{\nabla\varphi^\perp}{|\nabla\varphi|}\sigma_\tau dB_{\tau,t}\right) \\ &- \frac{A_i dt}{2|\nabla\varphi|^2} + \frac{\sigma_n F_i dt}{|\nabla\varphi|} + \frac{\sigma_\tau G_i dt}{|\nabla\varphi|}. \end{aligned}$$

Stochastic filtering for curve tracking

Definition of the transportation motion field

- Transportation motion field

$$v^*(x,t)dt = \frac{1}{\Delta t}(x - E(\psi^k(x,k)|\mathcal{C}_k)),\ \forall t \in [k,k+1],$$

Considering the particle approximation this velocity field is computed as:

$$v^*(x,t)dt = \frac{1}{\Delta t}\left(x - \frac{1}{N}\sum_{i=1}^{N} w^{(i)}\psi^{k,(i)}(x,k)\right), \tag{1}$$

for $t \in [k,k+1]$, with $\psi^{k,(i)}$ auxiliary function associated to particle $\varphi_k^{(i)}$ and $w_k^{(i)}$ its importance weight.

Stochastic filtering for curve tracking

Likelihood definition

Likelihood that depends on the similarity between photometric distributions inside the curve at times $t = 0$ and $t = k$.

$$p(\mathbf{y}_t|\mathcal{C}_t^{(i)}) \propto \exp\{-\lambda d(h_0, h_k^{(i)})\}, \tag{2}$$

d: Hellinger distance between h_0 the reference interior histogram at time 0 and $h_k^{(i)}$ the histogram associated to the i-th surface sample at time k

$$d(p,q) = \left(1 - \sum_{j \in X} \sqrt{p(j)q(j)}\right)^{1/2}.$$

Stochastic filtering for curve tracking

Results

Convective cell tracking

Stochastic filtering for curve tracking

Results

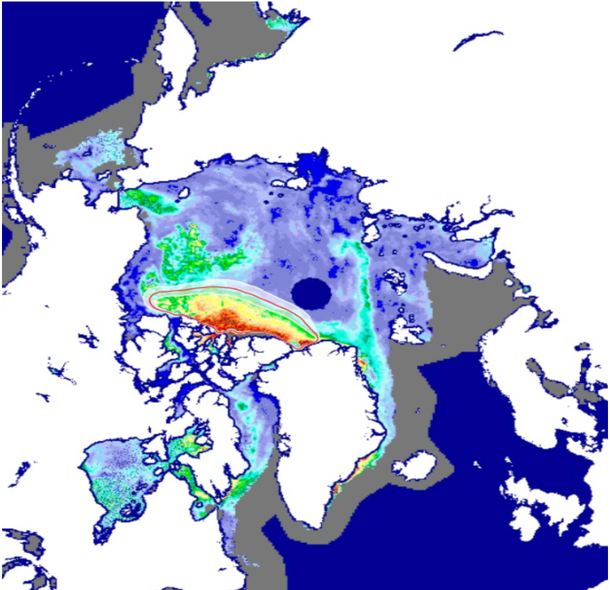
Convective cell: velocity field applied

Stochastic filtering for curve tracking

Results

Convective cell: curve points trajectories

Stochastic filtering for curve tracking



Variational assimilation

Principle

- Initial condition + dynamical law

$$\frac{\partial \boldsymbol{\mathcal{X}}}{\partial t}(x,t) + \mathbb{M}(\boldsymbol{\mathcal{X}}(x,t)) = p(x,t) \tag{3}$$

$$\boldsymbol{\mathcal{X}}(x,t_0) = \boldsymbol{\mathcal{X}}_0(x) + \epsilon_n(x), \tag{4}$$

- + observations

$$\boldsymbol{\mathcal{Y}}(x,t) = \mathbb{H}(\boldsymbol{\mathcal{X}}(x,t)) + \epsilon_o(x,t) \tag{5}$$

$\boldsymbol{\mathcal{X}}_0$ initial condition (t_0) and $(\mathbf{p}(\mathbf{t}), \boldsymbol{\epsilon}_\mathbf{n})$ control variables.

- Minimize w.r.t. (p, ϵ_n)

$$J(p,\epsilon_n) = \int_{t0}^{t_f} \|\boldsymbol{\mathcal{Y}}(x,t) - \mathbb{H}(\boldsymbol{\mathcal{X}}(x,t))\|_R^2 + \int_{t0}^{t_f} \|p(x,t)\|_Q^2 + \|\epsilon_n\|_B^2$$

Variational assimilation

Functional gradient

- Introduction of an adjoint variable

$$\begin{cases} -\dfrac{\partial \boldsymbol{\lambda}}{\partial t}(t) + (\partial_{\boldsymbol{\mathcal{X}}}\mathbb{M})^{*}\boldsymbol{\lambda}(t) = (\partial_{\boldsymbol{\mathcal{X}}}\mathbb{H})^{*}R^{-1}(\boldsymbol{\mathcal{Y}} - \mathbb{H}(\boldsymbol{\mathcal{X}}))(t) \\ \boldsymbol{\lambda}(t_f) = 0, \end{cases}$$

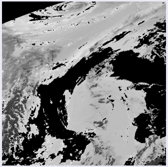
- Functional gradient

$$\begin{cases} \partial_p J = Q^{-1}(\partial_t \boldsymbol{\mathcal{X}} + \mathbb{M}(\boldsymbol{\mathcal{X}})) - \lambda, \\ \partial_{\epsilon_n} J = \lambda(t_0) + B^{-1}(\boldsymbol{\mathcal{X}}(t_0) - X_0) \end{cases}$$

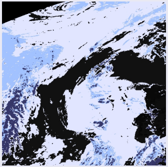
Variational assimilation for atmospheric layer motion tracking (Papadakis 07, Corpetti et al. Tellus 09)

Context: multi-layer imagery

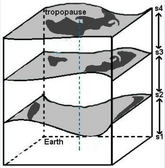
- Sparse images (clouds), inaccurate vertical coordinates



EUMETSAT
Top of cloud pressure p



EUMETSAT
Top of cloud classification $\{C^k\}$



Sparse images $(h^1_{obs}, h^2_{obs}, h^3_{obs})(t)$
for a 3 layer decomposition

Layer surfaces s^k

- cloud classified in $k-th$ layer
- s^k defined by clouds altitude in C^k

Sparse pressure differences h^k_{obs}

$$h^k_{obs} = \begin{cases} \overline{p}(s^k) - p(x,y,s^{k+1}) & \textit{if } (x,y) \in C^{k+1} \\ 0 & \textit{else}. \end{cases}$$

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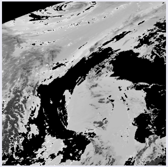

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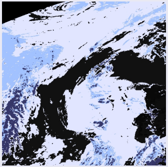

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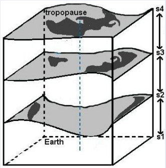

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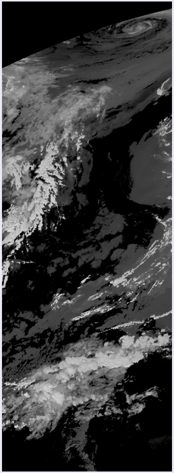
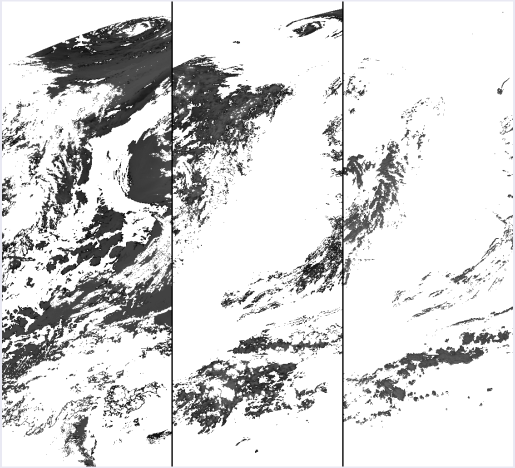
Variational assimilation for atmospheric layer motion tracking (Papadakis 07, Corpetti et al. Tellus 09)

Example of pressure differences image observations h^k_{obs}





Variational assimilation for atmospheric layer motion tracking (Papadakis 07, Corpetti et al. Tellus 09)

Dynamical model: simplified divergence and vorticity multi-layer shallow water model

$$\left\{ \begin{array}{l} \omega_t^k + \mathbf{v}^k \cdot \boldsymbol{\nabla} \omega^k + (\omega^k + f^\phi) D^k - \nu_{\mathcal{T}} \Delta \omega^k = \nu_1(D^k) \\ D_t^k + \mathbf{v}^k \cdot \boldsymbol{\nabla} D^k + (D^k)^2 - \nu_{\mathcal{T}} \Delta D^k = \nu_2(D^k, h^k) \end{array} \right. .$$

Motion field from Biot Savart law

$$\begin{aligned} \mathbf{v}^k &= \underbrace{\nabla^\perp G * \omega^k}_{\mathbf{v}_{sol}^k} + \underbrace{\nabla G * D^k}_{\mathbf{v}_{irr}^k} + \mathbf{v}_{har}^k \\ &= \underbrace{\left[\nabla^\perp G *, \nabla G *\right]}_{\mathbb{H}_G} \underbrace{\begin{bmatrix} \omega^k \\ D^k \end{bmatrix}}_{\text{state variable } \boldsymbol{\mathcal{X}}} + \mathbf{v}_{har}^k \end{aligned}$$

Variational assimilation for atmospheric layer motion tracking (Papadakis 07, Corpetti et al. Tellus 09)

Observations: optical-flow constraint equation (OFCE)

- Shallow water mass conservation

$$\frac{\partial h^k_{obs}}{\partial t} + \nabla h^k_{obs} \cdot v^k + h^k_{obs} \mathrm{div} v^k \approx 0$$

- Constant field v^k within a spatial neighborhood

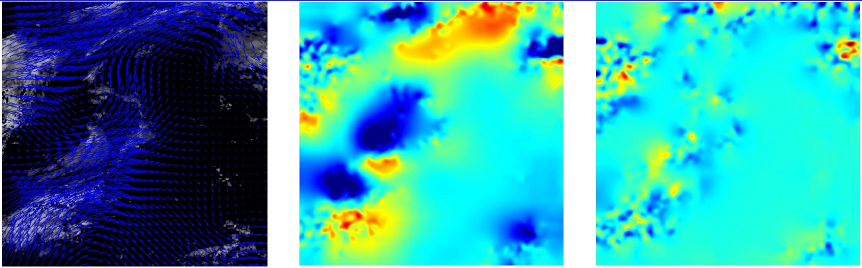
  (K_{δ_x} : Gaussian kernel)

$$K_{\delta_x} * \left(\frac{\partial h^k_{obs}}{\partial t} + \nabla h^k_{obs} \cdot \mathbf{v}^k + h^k_{obs} \mathrm{div} \mathbf{v}^k \right) \approx 0,$$

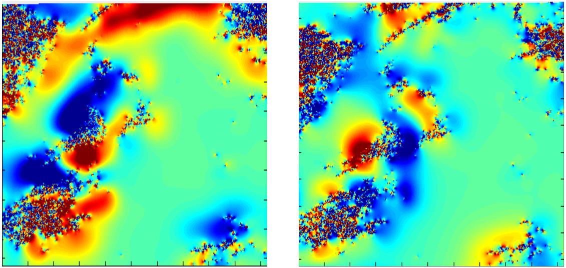
i.e. observation operator definition $\boldsymbol{\mathcal{Y}} = \mathbb{H}(\boldsymbol{\mathcal{X}})$ with $\mathbf{v}^k = \mathbb{H}_G \boldsymbol{\mathcal{X}}$ ($\boldsymbol{\mathcal{X}}$ is vorticity and divergence) :

$$\begin{cases} \boldsymbol{\mathcal{Y}} & = K_{\delta_x} * \frac{\partial h^k_{obs}}{\partial t} \\ \mathbb{H} & = -\left(K_{\delta_x} * \nabla h^k_{obs}\right)^T \mathbb{H}_G - \left(K_{\delta_x} * h^k_{obs}\right) [1 \quad 0] \end{cases}$$

Experiments on METEOSAT image sequence

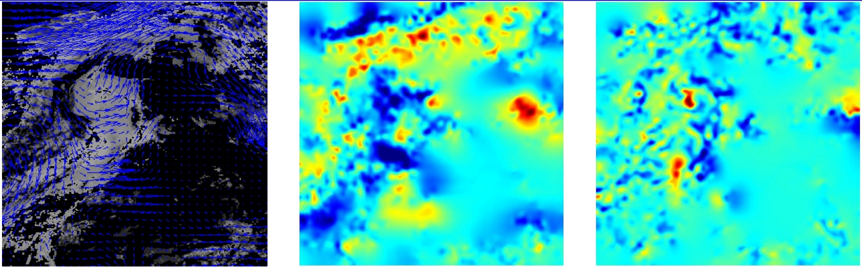


estimates (wind, vorticity & divergence of intermediate layer)

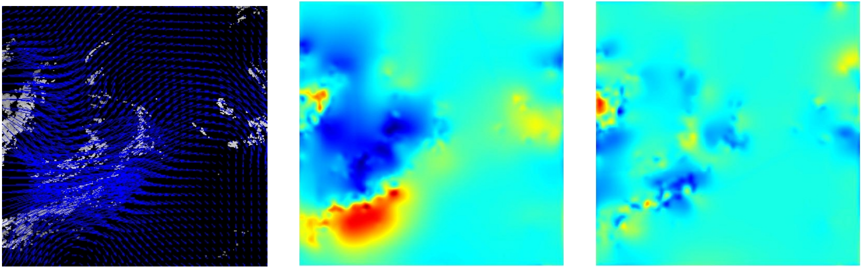


comparison with frame-to-frame motion estimation (vorticity & divergence)

Experiments on METEOSAT image sequence



estimates (wind, vorticity & divergence of lower layer)



estimates (wind, vorticity & divergence of higher layer)

Real sequence: vince (1/4)

- Cyclone (9^{th} october 2005)
- Infrared data
- Sequence of 116 images

Real sequence: vince (2/4)

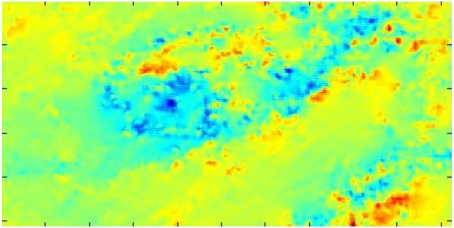
- Cyclone (9^{th} october 2005)
- Visible data
- Sequence of 116 images

Real sequence: vince (3/4)

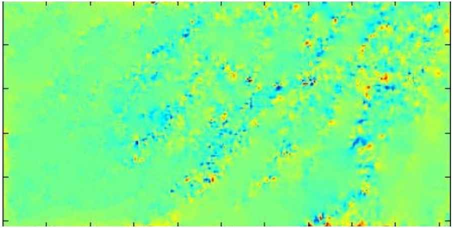
- Cyclone (9^{th} october 2005)
- Velocities obtained with the visible channel superimposed to IR data
- Sequence of 116 images

Real sequence: vince (4/4)

- Comparison with fluid dedicated optical-flow estimator (without temporal consistency)



Assimilated vorticity



No temporal consistency

Data assimilation

Stochastic filtering

- Recursive technique
- Probability distribution
- Adequate stochastic formulation of the dynamics
- Appropriate noise modeling

Variational assimilation

- Deterministic batch framework
- Adapted to state space of great dimension
- Require to built the adjoint of the tangent linear dynamics
- Intrinsic linearization

Conclusions

What did we do ?

- Explorations of tracking / assimilation techniques for time resolved flow images
- Very good performances for direct methods
- Process for “learning” dynamics from image data
- Specification of dynamics under uncertainty

But how far can we go ?

- Experiments mainly on 2D or 2D 1/2 flows
- Necessity to go toward 3D and more complex dynamics
- May have big potential applications for geo-physical applications